Effect of radiative loss on pulses in periodically inhomogeneous birefringent optical fibers

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(Received 19 December 1997)

The evolution of coupled pulses in a nonlinear birefringent optical fiber with a periodic modulation of the group velocity birefringence is considered. By using a trial function consisting of coupled pulses with variable parameters in the two modes in an averaged Lagrangian, ordinary differential equations for the pulse parameters are obtained. Furthermore, by considering linearized equations the effect of the dispersive radiation shed as the pulses evolve is calculated and the ordinary differential equations are augmented to include mass and momentum loss due to dispersive radiation. It is found that the inclusion of this dispersive radiation is necessary in order to obtain good agreement with full numerical solutions of the governing equations. [S1063-651X(98)07706-X]

PACS number(s): 42.81.Dp, 42.81.Gs

I. INTRODUCTION

The use of solitons as information carriers in optical communication systems was first proposed by Hasegawa and Tappert [1]. Since this time there has been extensive research on the dynamics of soliton propagation in nonlinear optical fibers. Much of this research has centered around the coupled solitons that exist in birefringent nonlinear optical fibers since birefringence can affect the propagation of solitons in long distance communication systems [2]. As well as having detrimental effects, birefringence can be used to advantage in all-optical devices such as nonlinear directional couplers [3]. Real optical fibers are of course nonuniform and the effect of this nonuniformity on soliton propagation needs to be investigated. The variation of fiber properties along a real optical fiber can be complicated and studies to date have chosen to consider either of two special cases, a periodic variation or a random variation. Soliton propagation in a nonuniform, nonbirefringent optical fiber, which is governed by the nonlinear Schrödinger (NLS) equation, has been investigated for a periodic variation of the dispersion by Gordon [4], Abdullaev *et al.* [5], Malomed *et al.* [6], and Abdullaev and Caputo [7] and for a random variation of the dispersion by Ueda and Kath [8] and Abdullaev and Caputo [7]. The extension to a birefringent fibre with a periodic variation of the birefringence parameter was made by Malomed and Smyth [9] using the chirp variational method of Anderson [10]. In the case of a birefringent fibre with a periodic birefringence modulation it was found that resonances between the forcing due to the periodic birefringence and the amplitude or position oscillations of the pulses could exist. However, the work of Malomed and Smyth did not take account of the dispersive radiation shed as the pulses evolve. It is expected that this radiation will have a significant effect on the pulse evolution, particularly when a resonance between the pulse oscillations and the forcing exists. It is the determination of the effect of this shed radiation that is the subject of the present work.

In the case of the NLS equation, Kath and Smyth [11] developed an approximate method to determine the effect of the dispersive radiation shed by a pulse as it evolves from an initial condition to a soliton. This method assumed that the pulse evolves with the sech profile of a NLS soliton, but with its parameters such as amplitude and width depending on the distance z down the fiber. Ordinary differential equations for these parameters were then determined via the Lagrangian for the NLS equation. The effect of the dispersive radiation shed as the pulse evolves was then added to these variational equations by determining an appropriate solution of the linearized NLS equation. Solutions of these variational equations with the effect of dispersive radiation added were found to be in excellent agreement with full numerical solutions of the NLS equation. The method of Kath and Smyth has been extended to model a nonlinear twin-core fiber (a type of all-optical switch) [12], coupled pulse propagation in a (uniform) birefringent optical fiber [13] and the stability of coupled solitons in a birefringent optical fiber [14]. In all these extensions it was found that there was very good agreement with full numerical solutions of the relevant governing equations. In previous studies, approximate equations governing pulse evolution in nonlinear optical fibers had been obtained using the chirp variational method of Anderson [10]. However, this method does not include the radiation shed by the pulses as they evolve. It has been found that by including the effect of this radiation, the method of Kath and Smyth yields significantly better agreement with full numerical solutions.

In the present work the method of Kath and Smyth [11] will be extended to study the evolution of coupled pulses in a nonlinear birefringent optical fiber whose birefringence parameter has a periodic variation down its length. It is found that the calculation of the shed radiation is particularly

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simple in this case as it is determined by the periodic form of the birefringence. This simplicity is similar to the case for the NLS equation with periodic dispersion [4]. Above a critical value of the amplitude of the periodic birefringence it is found that the pulses in the two modes split apart, as was found by Malomed and Smyth [9]. However, the critical values found by Malomed and Smyth are much lower than the values obtained from both the approximate equations of the present work and from full numerical solutions. This is due to the neglect of the radiation shed as the pulses evolve since the shed radiation acts as a damper on the oscillations and tends to keep the pulses together. Both the full numerical solution and the solution of the approximate equations derived in the present work show a broad trough in the critical forcing amplitude for pulse splitting in the vicinity of the resonances found by Malomed and Smyth. It is found that the inclusion of the dispersive radiation leads to good agreement between solutions of the present approximate equations and full numerical solutions of the coupled NLS equations governing a birefringent fiber. Indeed by comparing with solutions of the equations of Malomed and Smyth it is shown that the inclusion of this radiation is necessary in order to obtain adequate agreement with full numerical solutions.

II. APPROXIMATE EQUATIONS

Let us assume that the nonlinearity in the optical fiber can be described by the Kerr effect. Then the nondimensional equations governing optical fibers with a nonuniform birefringence and with two distinct modes operating in the anomalous dispersion regime are the coupled nonlinear Schrödinger (NLS) equations [15]

$$i\,\frac{\partial u}{\partial z}+i\,\delta(z)\,\frac{\partial u}{\partial t}+\frac{1}{2}\,\frac{\partial^2 u}{\partial t^2}+(|u|^2+A|v|^2)u=0,$$

$$i\,\frac{\partial v}{\partial z}-i\,\delta(z)\,\frac{\partial v}{\partial t}+\frac{1}{2}\,\frac{\partial^2 v}{\partial t^2}+(|v|^2+A|u|^2)v=0. \qquad (1)$$

Here u and v are the complex valued envelopes of the two modes, t is the normalized reduced time, and z is the normalized spatial variable along the length of the fiber. The parameter A is the scaled nonlinear cross-phase-modulation coefficient and $\delta(z)$ is half the difference in the linear group velocity due to the linear birefringence. As a simple model of a fiber with varying group velocity birefringence, it is assumed that δ is a periodic function of distance z down the fiber

$$\delta(z)=\epsilon\,\sin(kz). \qquad (2)$$

This expression for $\delta(z)$ has no constant component since it can be factored out using a simple phase transformation. This model of a nonuniform fiber is, of course, extremely idealized. For a start, the assumed periodic variation of the birefringence is unrealistic for a real fiber. Furthermore, the z dependence of the other coefficients in the coupled NLS equations (1) have been neglected and, in the case of a periodically twisted fiber, a linear coupling term in the coupled NLS equations has been neglected [16]. In spite of all these simplifications, the study of the coupled NLS equations (1) with δ given by Eq. (2) is still worthwhile as the very simplicity of the equations allows the effect of the dispersive radiation shed as pulses evolve to be calculated. This then gives an idea of the effect of dispersive radiation in more physically realistic cases.

The coupled NLS system (1) has the Lagrangian

$$\begin{aligned}L=&\,i(uu_z^*-u^*u_z)+|u_t|^2-|u|^4+i\,\delta(z)(uu_t^*-u^*u_t)\\&+i(vv_z^*-v^*v_z)+|v_t|^2-|v|^4-i\,\delta(z)(vv_t^*-v^*v_t)\\&-2A|u|^2|v|^2,\end{aligned} \qquad (3)$$

where $*$ denotes the complex conjugate. To obtain an approximate solution of the coupled NLS equations (1) for evolving coupled pulses, the forms

$$u=\left(\eta_1\text{sech}\,\frac{t-y_1}{w_1}+ig_1\right)e^{i\sigma_1+iV_1(t-y_1)},$$

$$v=\left(\eta_2\text{sech}\,\frac{t-y_2}{w_2}+ig_2\right)e^{i\sigma_2+iV_2(t-y_2)} \qquad (4)$$

will be assumed for the modes u and v, as in [11–14]. Here η_i, w_i, y_i, V_i, σ_i, and g_i, $i=1,2$, are functions of the distance z along the fiber. The first terms in these expressions for u and v represent varying solitary wave pulses. The second terms represent the effect of the (low frequency) radiation in the vicinity of the pulses [11]. These second terms are assumed to have no t dependence for two reasons. The first is that full numerical solutions of the coupled NLS equations (1) show that the radiation in the vicinity of the pulses has essentially constant magnitude, as in [11–14]. The second is that the perturbed inverse scattering solution of Gordon [4] for a near-soliton initial condition for the NLS equation shows that the dispersive radiation in the vicinity of the evolving soliton is of low frequency [11]. The dispersive radiation then forms shelves under the pulses, as has been observed in experimental situations [17]. Since from numerical solutions it is observed that the dispersive radiation has small amplitude relative to the pulses, it will be assumed that $|g_i|\ll\eta_i$, $i=1,2$. The final point to note about the pulse forms (4) is that the dispersive radiation cannot continue to be independent of t away from the pulses. As in [11–14], it is therefore assumed that the form of the dispersive radiation holds for $-\ell_1/2<t<\ell_1/2$ for mode u and $-\ell_2/2<t<\ell_2/2$ for mode v. The form of the dispersive radiation outside these intervals will be discussed in the next section. Furthermore, the values of ℓ_1 and ℓ_2 will be found by examining the yet to be derived pulse equations.

The trial functions (4) are now substituted into the Lagrangian (3), from which the averaged Lagrangian

$$\mathcal{L}=\int_{-\infty}^{\infty}L\,dt \qquad (5)$$

is obtained as

$$\mathcal{L}=4\eta_1^2w_1(\sigma_1'-V_1y_1')+2\pi\eta_1w_1g_1'-2\pi w_1g_1\eta_1'-2\pi\eta_1g_1w_1'+2\ell_1g_1^2(\sigma_1'-V_1y_1')+\frac{2}{3}\frac{\eta_1^2}{w_1}+2V_1^2\eta_1^2w_1-\frac{4}{3}\eta_1^4w_1$$
$$+4\delta V_1\eta_1^2w_1-2A\eta_1^2\eta_2^2I_1+4\eta_2^2w_2(\sigma_2'-V_2y_2')+2\pi\eta_2w_2g_2'-2\pi w_2g_2\eta_2'-2\pi\eta_2g_2w_2'+2\ell_2g_2^2(\sigma_2'-V_2y_2')+\frac{2}{3}\frac{\eta_2^2}{w_2}$$
$$+2V_2^2\eta_2^2w_2-\frac{4}{3}\eta_2^4w_2-4\delta V_2\eta_2^2w_2. \tag{6}$$

Here the integral I_1 is

$$I_1=\int_{-\infty}^{\infty}\operatorname{sech}^2\frac{t-y_1}{w_1}\operatorname{sech}^2\frac{t-y_2}{w_2}\,dt. \tag{7}$$

In calculating the averaged Lagrangian (6), the only terms of $O(g_1^2)$ and $O(g_2^2)$ that have been retained are those that will be found to contribute to mass conservation. Any other second order terms make a negligible contribution to the resulting variational equations and have been neglected.

Taking variations of the averaged Lagrangian (6) with respect to the parameter results, on some algebraic manipulation, in the following equations governing the evolution of the pulses through the nonuniform fiber

$$\frac{d}{dz}(\eta_1w_1)=\frac{\ell_1g_1}{\pi}\left[\eta_1^2-\frac{1}{2}w_1^{-2}+A\frac{\eta_2^2}{w_1^2}(w_1I_1-I_2)\right], \tag{8}$$

$$\frac{d\sigma_1}{dz}-\frac{1}{2}V_1\frac{dy_1}{dz}=\eta_1^2-\frac{1}{2}w_1^{-2}-\delta(z)V_1+A\frac{\eta_2^2}{w_1^2}(w_1I_1-I_2), \tag{9}$$

$$\frac{dg_1}{dz}=-\frac{2}{3\pi}\eta_1(\eta_1^2-w_1^{-2})-\frac{A\eta_1\eta_2^2}{\pi w_1^2}(w_1I_1-2I_2), \tag{10}$$

$$\frac{d}{dz}(2\eta_1^2w_1+\ell_1g_1^2)=0, \tag{11}$$

$$\frac{d}{dz}[(2\eta_1^2w_1+\ell_1g_1^2)V_1]=-\frac{2A\eta_1^2\eta_2^2}{w_2}I_3, \tag{12}$$

$$\frac{dy_1}{dz}=V_1+\delta(z), \tag{13}$$

plus symmetric equations for the parameters of mode v. The integrals I_1 and I_2 in these variational equations are defined by

$$I_2=\int_{-\infty}^{\infty}(t-y_1)\operatorname{sech}^2\frac{t-y_1}{w_1}\operatorname{sech}^2\frac{t-y_2}{w_2}\tanh\frac{t-y_1}{w_1}\,dt \tag{14}$$

and

$$I_3=\int_{-\infty}^{\infty}\operatorname{sech}^2\frac{t-y_1}{w_1}\operatorname{sech}^2\frac{t-y_2}{w_2}\tanh\frac{t-y_2}{w_2}\,dt. \tag{15}$$

The coupled NLS equations (1) have the mass conservation equation

$$\frac{\partial}{\partial z}\int_{-\infty}^{\infty}|u|^2\,dt=0, \tag{16}$$

the momentum conservation equation

$$i\frac{\partial}{\partial z}\int_{-\infty}^{\infty}(u^*u_t-uu_t^*)dt=-2A\int_{-\infty}^{\infty}|u|^2\frac{\partial}{\partial t}(|v|^2)dt, \tag{17}$$

and the energy conservation equation

$$\frac{\partial}{\partial z}\int_{-\infty}^{\infty}[i(|u_t|^2-|u|^4+|v_t|^2-|v|^4-2A|u|^2|v|^2)+\delta(z)(u^*u_t-uu_t^*)-\delta(z)(v^*v_t-vv_t^*)]dt$$
$$=\delta'(z)\int_{-\infty}^{\infty}[(u^*u_t-uu_t^*)-(v^*v_t-vv_t^*)]dt. \tag{18}$$

Substituting the trial functions (4) into the mass conservation equation (16) gives the variational equation (11). Similarly the variational equation (12) can be shown to express conservation of momentum on using the momentum conservation equation (17). Finally, substituting the trial functions (4) into the energy conservation equation (18) yields

$$\frac{dH}{dz}=\frac{d}{dz}\left[\frac{2}{3}\frac{\eta_1^2}{w_1}+V_1^2(2\eta_1^2w_1+\ell_1g_1^2)-\frac{4}{3}\eta_1^4w_1+\frac{2}{3}\frac{\eta_2^2}{w_2}+V_2^2(2\eta_2^2w_2+\ell_2g_2^2)-\frac{4}{3}\eta_2^4w_2-2A\eta_1^2\eta_2^2I_1+2\delta V_1(2\eta_1^2w_1+\ell_1g_1^2)\right.$$
$$\left.-2\delta V_2(2\eta_2^2w_2+\ell_2g_2^2)\right]$$
$$=2V_1\delta'(z)(2\eta_1^2w_1+\ell_1g_1^2)-2V_2\delta'(z)(2\eta_2^2w_2+\ell_2g_2^2). \quad (19)$$

It is noted from this equation that the energy H is not conserved due to the variation in the birefringence δ with z.

For simplicity, in the present work we shall concentrate on the case of antisymmetric modes u and v, so that $\eta_1=\eta_2$, $w_1=w_2$, $\sigma_1=\sigma_2$, $V_1=-V_2$, $y_1=-y_2$, and $g_1=g_2$. This antisymmetric assumption allows the integrals I_1 (7), I_2 (14), and I_3 (15) to be evaluated exactly. Under this assumption these integrals can be shown to be

$$I_1=\frac{4w_1}{\sinh^2\zeta}[\zeta\coth\zeta-1],$$

$$I_2=\frac{w_1^2}{\sinh^2\zeta}[\zeta\coth\zeta-1],$$

$$-\frac{\zeta w_1^2}{\sinh^2\zeta}[3\coth\zeta+\zeta-3\zeta\coth^2\zeta], \quad (20)$$

and

$$I_3=\frac{2w_1}{\sinh^2\zeta}[3\zeta\coth^2\zeta-3\coth\zeta-\zeta],$$

where

$$\zeta=\frac{2y_1}{w_1}. \quad (21)$$

We shall now replace the mass conservation equation (11) by the energy conservation equation (19) in the set of equations governing the evolution of the pulses. The variational equations (8) to (13) plus the energy conservation equation are not all independent, so this can be done. Using the momentum conservation equation (12) and the antisymmetry assumption, the energy conservation equation (19) can be rewritten as

$$\frac{dH}{dz}=\frac{d}{dz}\left[\frac{4}{3}\frac{\eta_1^2}{w_1}-\frac{8}{3}\eta_1^4w_1+2(2\eta_1^2w_1+\ell_1g_1^2)V_1^2-2A\eta_1^2\eta_2^2I_1\right]=8A\delta(z)\frac{\eta_1^2\eta_2^2}{w_1}I_3. \quad (22)$$

With the assumed antisymmetry, the equations governing the evolution of the pulses in the nonuniform fiber are then (8)–(10), (12), and (22). These equations for constant δ are the same as those of [13] for pulse propagation in a birefringent fiber.

For a birefringent fiber, Kath and Smyth [13] showed that the length ℓ_1 of the shelves under the pulses could be determined from the requirement that the frequency of oscillation of the pulse amplitude η_1 as given by the approximate equations approaches the soliton oscillation frequency as a steady state is approached. This requirement gave

$$\ell_1=\frac{3\pi^2}{8\hat{\eta}\sqrt{1+A}} \quad (23)$$

when the pulses approached the coupled soliton solution of the coupled NLS equations (1) for $\delta=$const, where $\hat{\eta}$ is the amplitude of the steady coupled solitary waves. In the present case of a nonuniform fiber the pulses will not approach a steady state since the fiber is continuously varying. However, the shelf width expression (23) can still be used, with the steady state $\hat{\eta}$ replaced by the local pulse amplitude η_1. Therefore, in the present work, the shelf width will be taken to be

$$\ell_1=\frac{3\pi^2}{8\eta_1\sqrt{1+A}}. \quad (24)$$

Replacing the steady state amplitude by its local value allows the approximate equations to respond to the local fiber properties.

The system of equations governing the evolution of pulses in a nonuniform fiber is not complete, however, since the effect of the dispersive radiation shed by the pulses as they evolve has not been taken into account. This radiation is the subject of the next section.

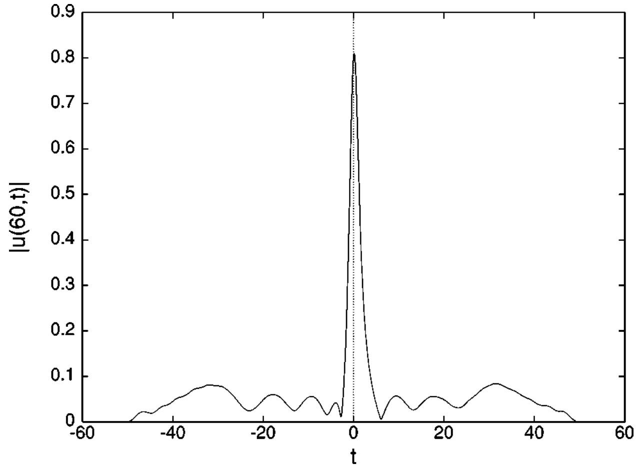

FIG. 1. Numerical solution for $|u|$ of the coupled NLS equations (1) at $z=60.0$ for $\epsilon=0.1$, $k=1.0$, $A=2/3$ and the initial conditions $\eta=1.0$ and $w=1.0/\sqrt{1+A}$.

III. DISPERSIVE RADIATION

As can be seen from Fig. 1, the amplitude of the dispersive radiation shed by the pulses is small compared with the pulse amplitude. Hence the shed dispersive radiation is governed by the linearized NLS equation

$$i\,\frac{\partial u}{\partial z}+i\,\delta(z)\,\frac{\partial u}{\partial t}+\frac{1}{2}\,\frac{\partial^2 u}{\partial t^2}=0. \tag{25}$$

In previous studies of the radiation shed by an evolving pulse [11–14] this radiation was calculated as the solution of an initial value problem for an appropriate linearized NLS equation. In the present case of a nonuniform fiber, this initial value type solution is not appropriate since the fiber is varying continuously, which causes the pulses to never settle to a steady state and so radiate continuously. Hence the appropriate solution of the linearized NLS equation (25) in the present context is that of a continuous source.

It can be easily verified that the linearized NLS equation (25) has the modulated traveling wave solution

$$u=Be^{i\beta t-(1/2)i\beta^2 z+i(\beta\epsilon/k)\cos(kz)}, \tag{26}$$

where B and β are constants. Let us assume for the moment that the properties of the pulses were constant. Then if we denote the height of the shelves below the evolving pulses by r, matching between the shelf and the shed radiation gives $B=r$. Since the radiation is being shed by the pulses, the phase velocity of the shed radiation at the end of the shelf must be equal to the pulse velocity, so that

$$\beta=2V_1 \tag{27}$$

[see the variational equation (13)]. These expressions for B and β are only strictly valid if the pulses have constant properties. However, let us assume that these expressions for B and β hold when the pulses are varying. This is equivalent to assuming that B and β are slowly varying, which is the case when ϵ is small, as in the present work.

The linearized NLS equation (25) has the mass and momentum conservation equations

$$i\,\frac{\partial}{\partial z}|u|^2+\frac{1}{2}\,\frac{\partial}{\partial t}[2i\,\delta(z)|u|^2+u^*u_t-uu_t^*]=0 \tag{28}$$

and

$$i\,\frac{\partial}{\partial z}(u^*u_t-uu_t^*)+\frac{1}{2}\,\frac{\partial}{\partial t}[2i\,\delta(z)(u^*u_t-uu_t^*)+u^*u_{tt}+uu_{tt}^*-2|u_t|^2]=0, \tag{29}$$

respectively. Integrating the mass conservation equation (28) from the edge of the shelf $t=y_1+\ell_1/2$ to $t=\infty$ gives the mass loss to the dispersive radiation traveling to the right as

$$\frac{d}{dz}\int_{y_1+\ell_1/2}^{\infty}|u|^2dt=V_1r^2 \tag{30}$$

on using the form (26) for the dispersive radiation. Similarly integrating the mass conservation equation from $t=-\infty$ to $t=y_1-\ell_1/2$ gives the mass loss to dispersive radiation traveling to the left as

$$\frac{d}{dz}\int_{-\infty}^{y_1-\ell_1/2}|u|^2dt=-V_1r^2. \tag{31}$$

The momentum conservation equation (29) can be integrated in the same way to give the momentum loss to dispersive radiation traveling to the right as

$$i\,\frac{d}{dz}\int_{y_1+\ell_1/2}^{\infty}(u^*u_t-uu_t^*)dt=-4V_1^2r^2 \tag{32}$$

and the momentum loss to radiation traveling to the left as

$$i\,\frac{d}{dz}\int_{-\infty}^{y_1-\ell_1/2}(u^*u_t-uu_t^*)dt=4V_1^2r^2. \tag{33}$$

These mass and momentum loss expressions can then be added to the variational equations of the previous section to complete the approximate equations governing the evolution of the pulses. Before this is done, however, care must be taken with the form (26) for the shed radiation. This radiation has phase velocity $V_1+\delta$. Now the group velocity for the radiation (26) is $c_g=2V_1+\delta$. Hence for $V_1>0$, the radiation is shed to the right of the pulse u and the mass and momentum loss expressions (30) and (32) are the appropriate ones. By the same reasoning, the mass and momentum loss expressions (31) and (33) are the appropriate ones for $V_1<0$. This unsymmetric nature of the mass and momentum loss can be seen in Fig. 1.

As in [11–14], the appropriate mass loss expression (30) or (31) is added to the variational equations of the previous section, so that the equation for g_1 (10) becomes

$$\frac{dg_1}{dz}=-\frac{2}{3\pi}\,\eta_1(\eta_1^2-w_1^{-2})-\frac{A\,\eta_1\eta_2^2}{\pi w_1^2}(w_1I_1-2I_2)-2\alpha g, \tag{34}$$

where

$$\alpha = \frac{3\eta_1 |V_1| \sqrt{1+A}}{16}. \tag{35}$$

Adding the appropriate momentum loss expression (32) or (33) to the momentum equation (12) gives the final momentum conservation equation as

$$\frac{d}{dz}[(2\eta_1^2 w_1 + \ell_1 g_1^2)V_1] = -\frac{2A\eta_1^2\eta_2^2}{w_2} I_3 - 2\theta V_1^2 r^2, \tag{36}$$

where $\theta = 1$ if $V_1 > 0$ and $\theta = -1$ if $V_1 < 0$, as discussed in the previous paragraph.

The final quantity needed, the height r of the shelf under the pulses, can be found from the work of Kath and Smyth [13] on pulse evolution in birefringent fibers as

$$r^2 = \frac{3\eta_1\sqrt{1+A}}{8}(2\eta_1^2 w_1 + \ell_1 g_1^2). \tag{37}$$

The full set of equations governing the evolution of the pulses, including mass and momentum loss to dispersive radiation, has now been derived, these equations being (8), (9), (13), (22), (34), and (36). In the next section solutions of these approximate equations will be compared with full numerical solutions of the coupled NLS equations (1) and solutions of the chirp equations of [9].

IV. RESULTS

In the comparisons of this section the numerical solutions of the coupled NLS equations (1) will be obtained using the pseudospectral method of Fornberg and Whitham [18] and the approximate equations (8), (9), (13), (22), (34), and (36) will be solved using a fourth order Runge-Kutta method. Employing the chirp method of Anderson [10], Malomed and Smyth [9] found that for low values of ϵ the pulses undergo oscillations about each other, while above a critical value of ϵ, which depends on the initial energy of the pulses, the pulses split into a pair of simple NLS solitons. It was further shown that resonances between the internal oscillation frequencies of the pulses and the frequency of the periodic birefringence were possible. Near these resonances the critical value of ϵ, ϵ_{cr}, for pulse splitting was lower by up to an order of magnitude, again depending on the initial energy of the pulses. The same general picture is obtained in the present work. However, it is found that when the dispersive radiation shed by the pulses is taken into account, ϵ_{cr} is increased by an order of magnitude. This is because the shed radiation acts as a damping that tends to keep the pulses together. The present case of a fiber with periodic nonuniform birefringence therefore represents a situation in which the effect of dispersive radiation is crucial.

Figure 2 shows the critical value ϵ_{cr} of the amplitude of the periodic birefringence above which the pulses split as a function of the initial amplitude η_0 of the pulses. The initial condition used was a coupled soliton of width $w = 1/(\eta_0\sqrt{1+A})$ with the parameter values $k=1$ and $A = 2/3$ (the value for glass). Shown are the values of ϵ_{cr} as given by the full numerical solution of the coupled NLS equations (1) and by the solution of the present approximate equations. Below $\eta_0 = 0.9$ the critical value ϵ_{cr} increases rapidly. This is because for low initial amplitudes the radiation causes so much relative decay of the pulses that they have difficulty in escaping. It can be seen that both the approximate and full numerical solutions show a marked reduction in ϵ_{cr} between $\eta_0 = 0.9$ and $\eta_0 = 1.6$. This is due to resonances between the periodic birefringence and the internal oscillations of the coupled pulses. Malomed and Smyth [9] identified a resonance between the forcing and the positional oscillations of the pulses for

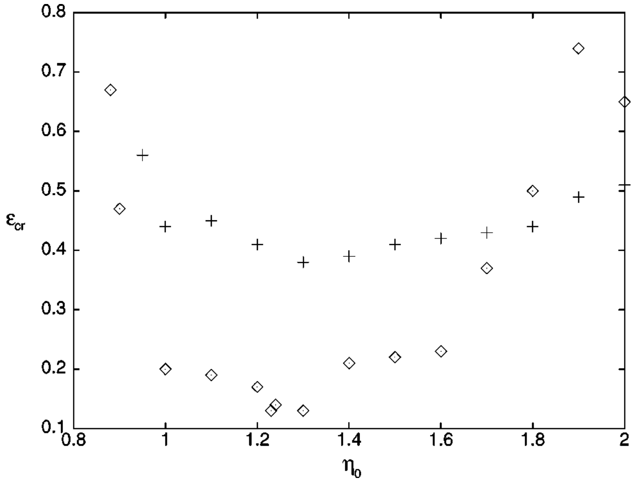

FIG. 2. Critical forcing amplitude for pulse separation ϵ_{cr} as a function of initial pulse amplitude η_0 for $w = 1.0/(\eta_0\sqrt{1+A})$, $k = 1.0$, and $A = 2/3$. Full numerical solution: $\diamond$; approximate equations: $+$.

$$(\eta_1^2 w_1)^{-4} = \frac{16}{15}A(1+A)^3 \tag{38}$$

and a resonance between the forcing and the width oscillations of the pulses for

$$(\eta_1^2 w_1)^{-2} = \frac{1}{\pi}(1+A)^2 \tag{39}$$

(note that $\eta_1^2 w_1$ is a conserved quantity when radiation is neglected, as in Malomed and Smyth [9]). These resonances occur at $\eta_0 = 0.96$ and $\eta_0 = 1.37$ respectively for the parameter values of Fig. 2. The large trough in Fig. 2 thus encompasses these two resonant values. Outside of this trough it can be seen that the agreement between the critical values of ϵ is quite good. However, inside the trough there is significant disagreement between the approximate and full numerical values. The reason for this is the chaotic nature of the oscillations of the pulse parameters as the critical value of ϵ is approached, particularly in the resonant trough for which chaotic oscillations set in for quite low values of ϵ.

While the present approximate equations do not yield good agreement for the values of ϵ_{cr} in the resonant trough, the agreement obtained in the present work is much better than that obtained by Malomed and Smyth [9]. Malomed and Smyth also found a resonant trough lying between about $\eta_0 = 0.8$ and $\eta_0 = 1.5$. However, the values of ϵ_{cr} in this trough were about 0.05, while outside the trough they found critical values of around 0.6 to 0.8. So by including radiation

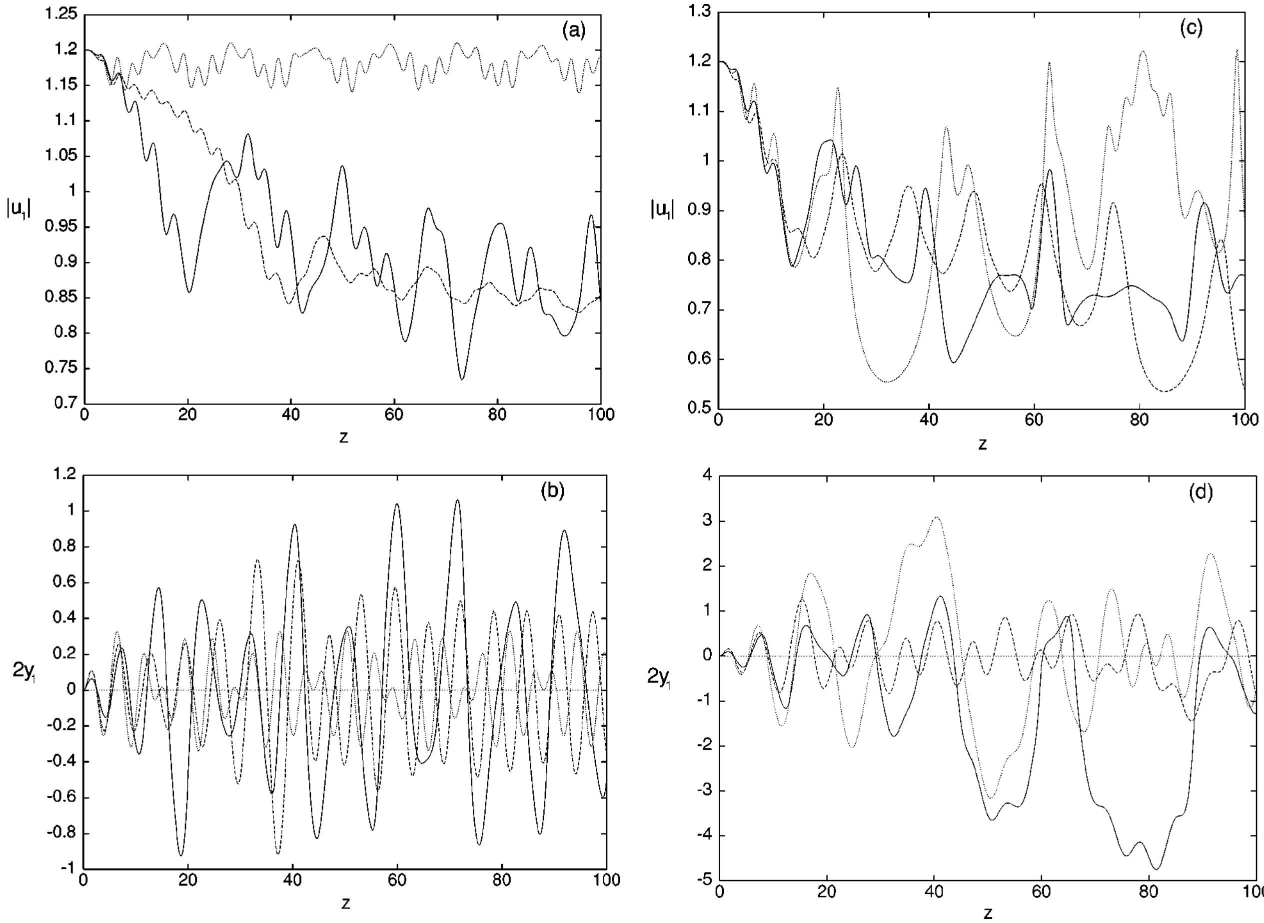

FIG. 3. Comparison between full numerical solution: ——; solution of present approximate equations: – – –; approximate equations of Malomed and Smyth: ----. The initial conditions are $\eta_0=1.2$ and $w=1.0/(\eta_0\sqrt{1+A})$ with $k=1.0$ and $A=2/3$. (a) Amplitude of pulse u_1 as a function of z for $\epsilon=0.1$. (b) Pulse separation $2y_1$ as a function of z for $\epsilon=0.1$. (c) Amplitude of pulse u_1 as a function of z for $\epsilon=0.15$. (d) Pulse separation $2y_1$ as a function of z for $\epsilon=0.15$.

the present approximation gives better agreement in the resonant trough, where the effects of radiation are expected to be more important.

Figure 3 shows amplitude and position comparisons for $\eta_0=1.2$ and $w_0=1/(\eta_0\sqrt{1+A})$ with $k=1$ and $A=2/3$. Shown are comparisons between the full numerical solution of the coupled NLS equations (1), the solution of the present approximate equations and the solution of the equations of Malomed and Smyth [9]. The initial condition $\eta_0=1.2$ lies in the middle of the resonant trough of Fig. 2. In Figs. 3(a) and 3(b) these comparisons are shown for $\epsilon=0.1$, which lies well below the critical forcing amplitude. It can be seen that even well below the critical forcing amplitude the full numerical solution shows oscillations that look chaotic, so that precise agreement is not expected with the approximate solution. However, it can be seen from Fig. 3(a) that while the exact details of the pulse amplitude oscillations are not given by the present approximate equations, the overall trend is well predicted, with good agreement with the rate at which the numerical amplitude decays. This is in contrast with the solution of the equations of Malomed and Smyth for which there is no amplitude decay due to the shed dispersive radiation not being taken into account. The position oscillations shown in Fig. 3(b) again look chaotic and the detailed agreement between the numerical and approximate solutions is not good. However, the overall trend of the position oscillations is reasonably well predicted by the present approximate equations.

Figures 3(c) and 3(d) show amplitude and position comparisons for $\epsilon=0.15$, which is approaching the critical forcing amplitude. The oscillations are now quite chaotic. However, as for the lower value $\epsilon=0.1$, the overall decay of the amplitude is well predicted by the present approximate equations. The amplitude as given by the approximate equations of Malomed and Smyth shows too large an oscillation and no decay. The separation of the pulses as given by the present approximate equations is not in good agreement with the full numerical solution. This is to be expected as the numerical position oscillations are very chaotic. Also the critical value of ϵ for $\eta_0=1.2$ as given by the approximate equations is above the numerical value, so good agreement is not expected near the critical for the position of the pulses.

Better agreement between the approximate and full numerical solutions is obtained when the initial condition lies away from the middle of the resonant trough. Such a case is shown in Fig. 4 where comparisons between the full numerical solution, the solution of the present approximate equations, and the approximate equations of Malomed and Smyth [9] are shown for the initial conditions $\eta_0=1.0$ and $w_0=1/(\eta_0\sqrt{1+A})$ with $\epsilon=0.1$, $k=1.0$, and $A=2/3$. Compari-

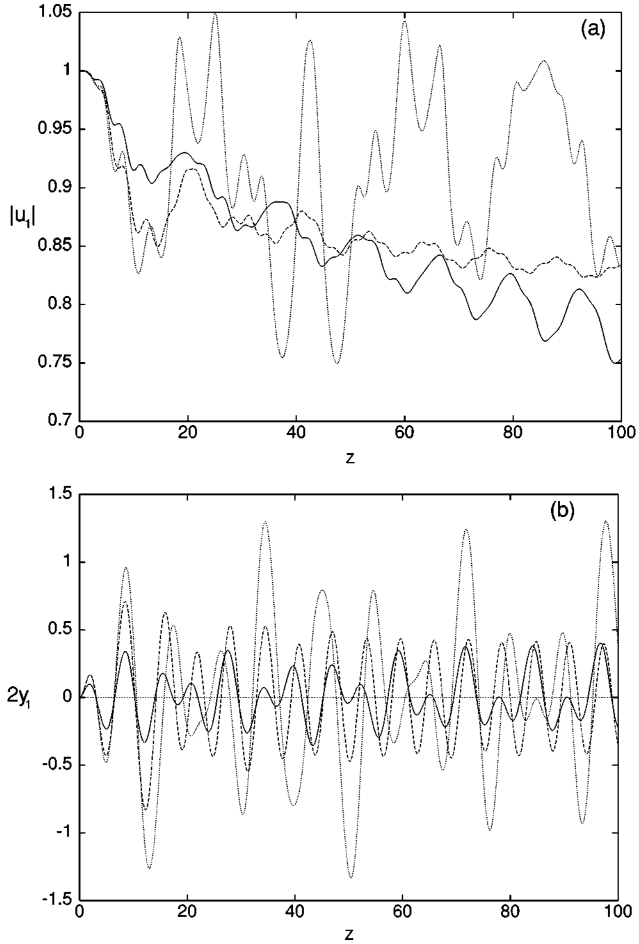

FIG. 4. Comparison between full numerical solution: —— ; solution of present approximate equations: — — —; approximate equations of Malomed and Smyth: ----. The initial conditions are $\eta_0=1.0$ and $w=1.0/(\eta_0\sqrt{1+A})$ with $k=1.0$, $\epsilon=0.10$, and $A=2/3$. (a) Amplitude of pulse u_1 as a function of z. (b) Pulse separation $2y_1$ as a function of z.

sons between the amplitude of the pulse u_1 and the separation of the pulses are shown. It can be seen that there is good agreement between the solution of the present approximate equations and the full numerical solution, in marked contrast to the solution of the approximate equations of Malomed and Smyth. If the effect of the shed radiation was neglected in the present approximate equations, so that $\alpha=0$ in Eq. (34) and the θ term in the momentum equation (36) was neglected, then the present equations would yield results similar to those for the equations of Malomed and Smyth, as shown in Fig. 4. Hence the inclusion of the dispersive radiation shed as the pulses evolve is critical to obtaining good agreement with the full numerical solution.

V. CONCLUSIONS

Approximate equations governing coupled pulse evolution in a nonlinear optical fiber with a periodically varying birefringence have been derived using an averaged Lagrangian. The effect of the dispersive radiation shed as the pulses evolve has been included by finding appropriate solutions of the linearized form of the coupled NLS equations governing the pulses. For high enough values of the amplitude of the periodic birefringence the pulses in the two modes split. Resonances between the periodic forcing and the internal oscillations of the pulses are possible and around these resonances the critical amplitude of the birefringence for pulse splitting is significantly lowered. Good agreement between the solutions of the approximate equations and full numerical solutions were obtained for parameter values away from resonance. Near resonance the oscillations of the pulse amplitude and position were found to be chaotic, so that good agreement between the approximate and numerical solutions was not obtained, or indeed expected. However, the rate of decay of the pulse amplitude was found to be well predicted by the approximate equations, even near resonance. By comparison with previous approximate solutions which did not include the effect of the dispersive radiation shed as the pulses evolve, it was shown that the inclusion of this radiation is necessary in order to obtain good agreement with full numerical solutions.

ACKNOWLEDGMENTS

N.F.S. would like to thank B.S. and S.S. for helpful discussions.

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